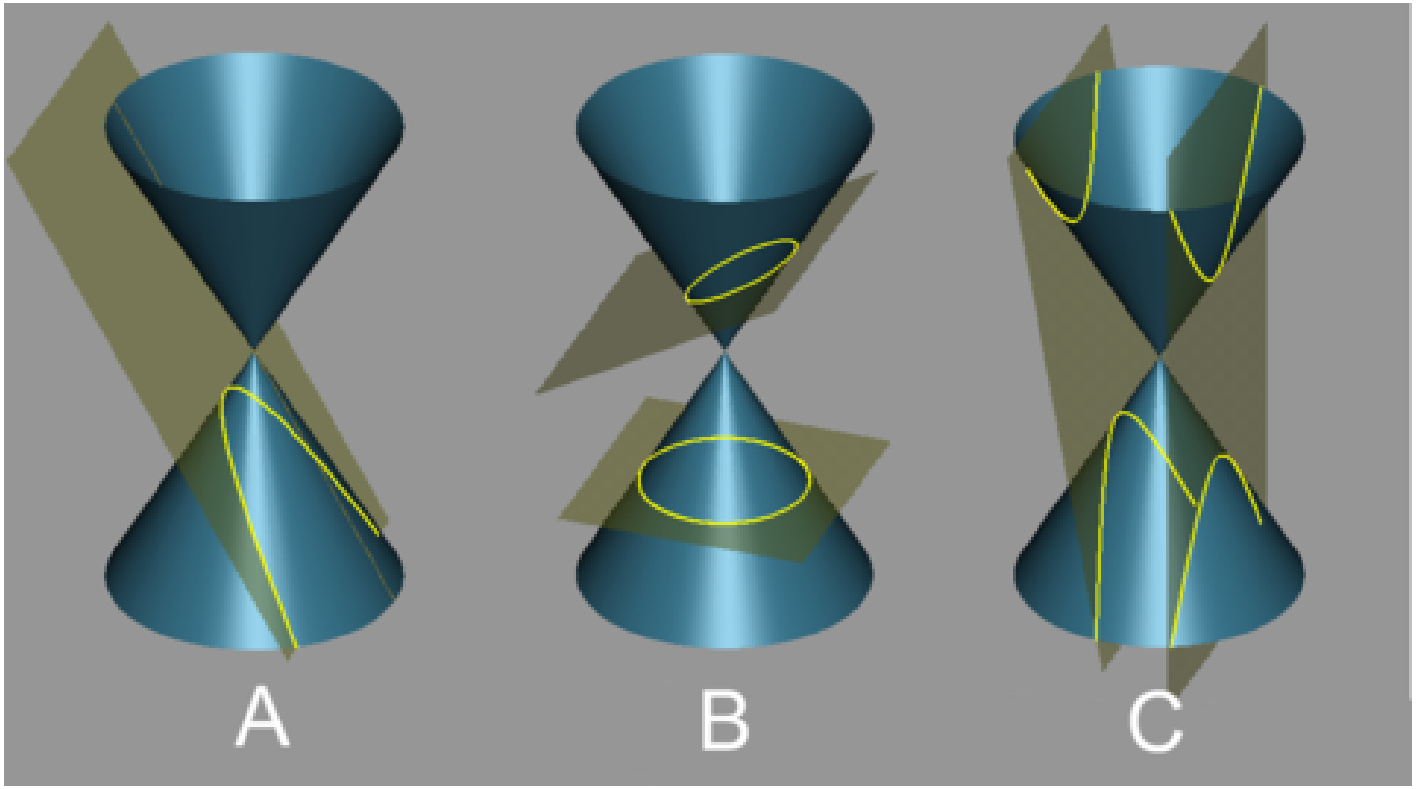
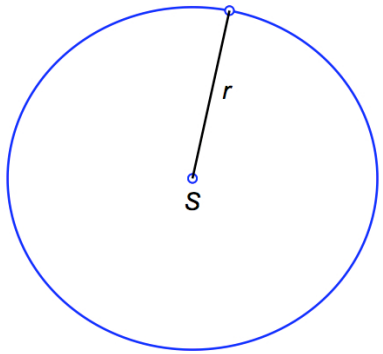
Kuželosečky



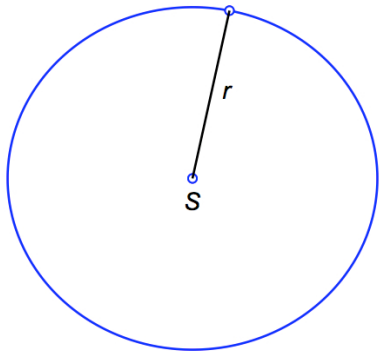
Kuželosečky, známe množiny bodov

- Kružnica
- Elipsa
- Hyperbola
- Parabola

Kružnica

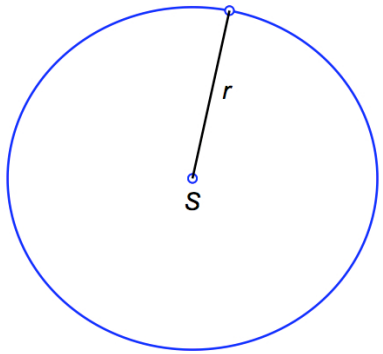


Kružnica



$$|XS| = r \Rightarrow (x-m)^2 + (y-n)^2 = r^2$$

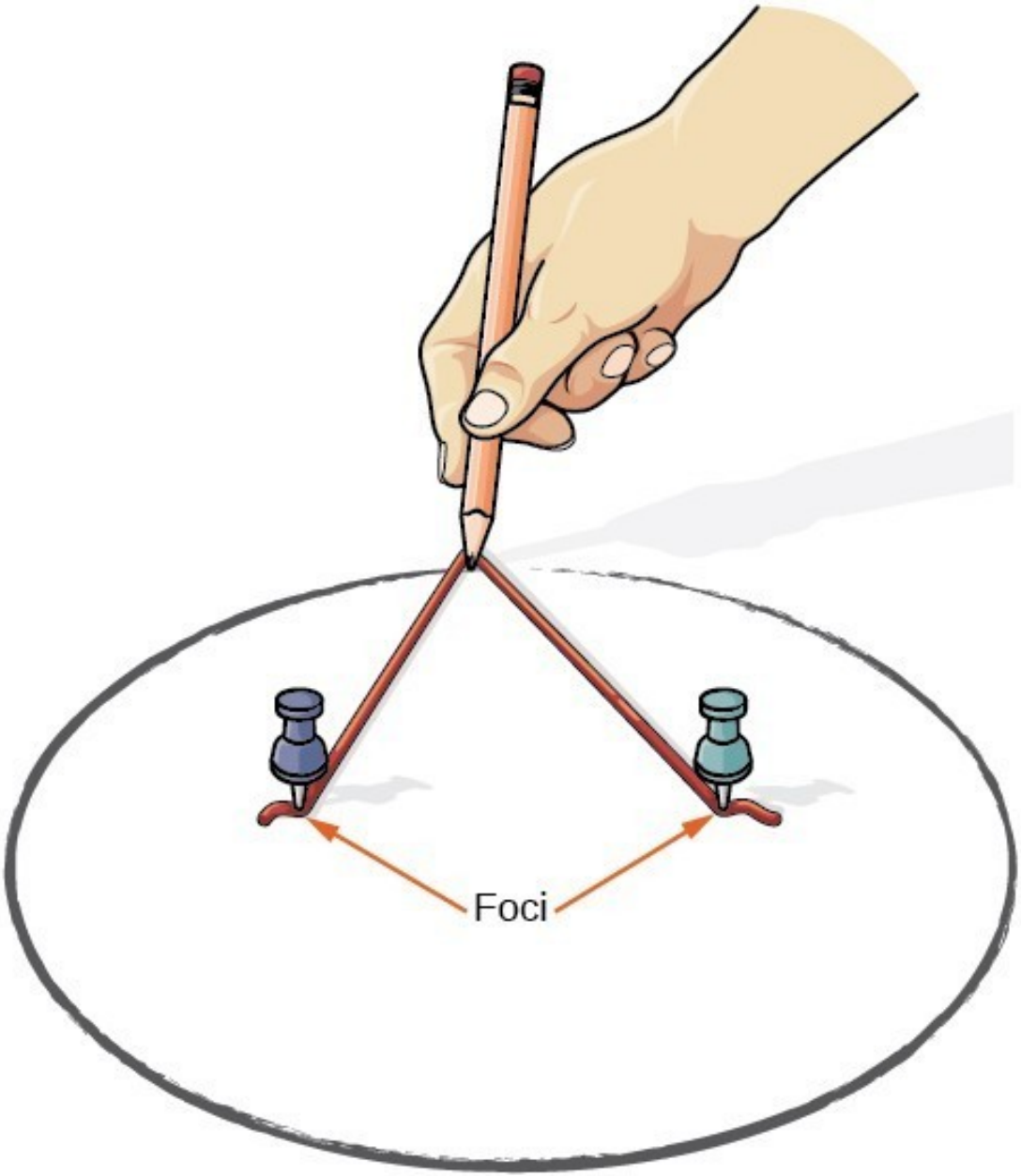
Kružnica



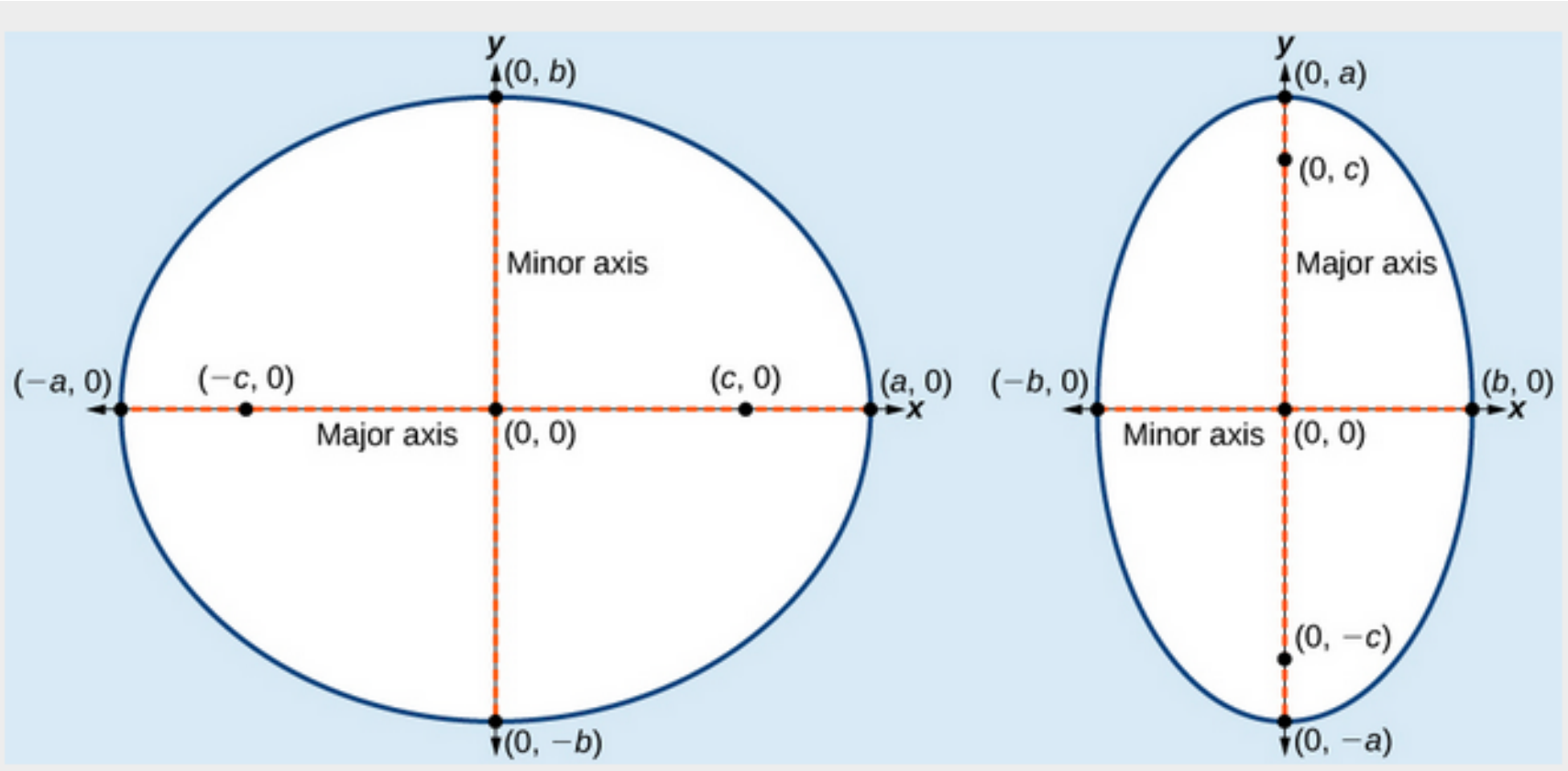
$$|XS| = r \Rightarrow (x-m)^2 + (y-n)^2 = r^2$$

Stred $S = (m, n)$, polomer r.

Elipsa



Elipsa



$$a^2 = e^2 + b^2.$$

(a) $|XE| + |XF| = 2a \Rightarrow \frac{(x-m)^2}{a^2} + \frac{(y-n)^2}{b^2} = 1,$

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Elipsa

(a) $|XE| + |XF| = 2a \Rightarrow \frac{(x-m)^2}{a^2} + \frac{(y-n)^2}{b^2} = 1,$

Stred $S = (m, n)$, hlavná poloos a,

(b) $|XE| + |XF| = 2a \Rightarrow \frac{(x-m)^2}{b^2} + \frac{(y-n)^2}{a^2} = 1,$

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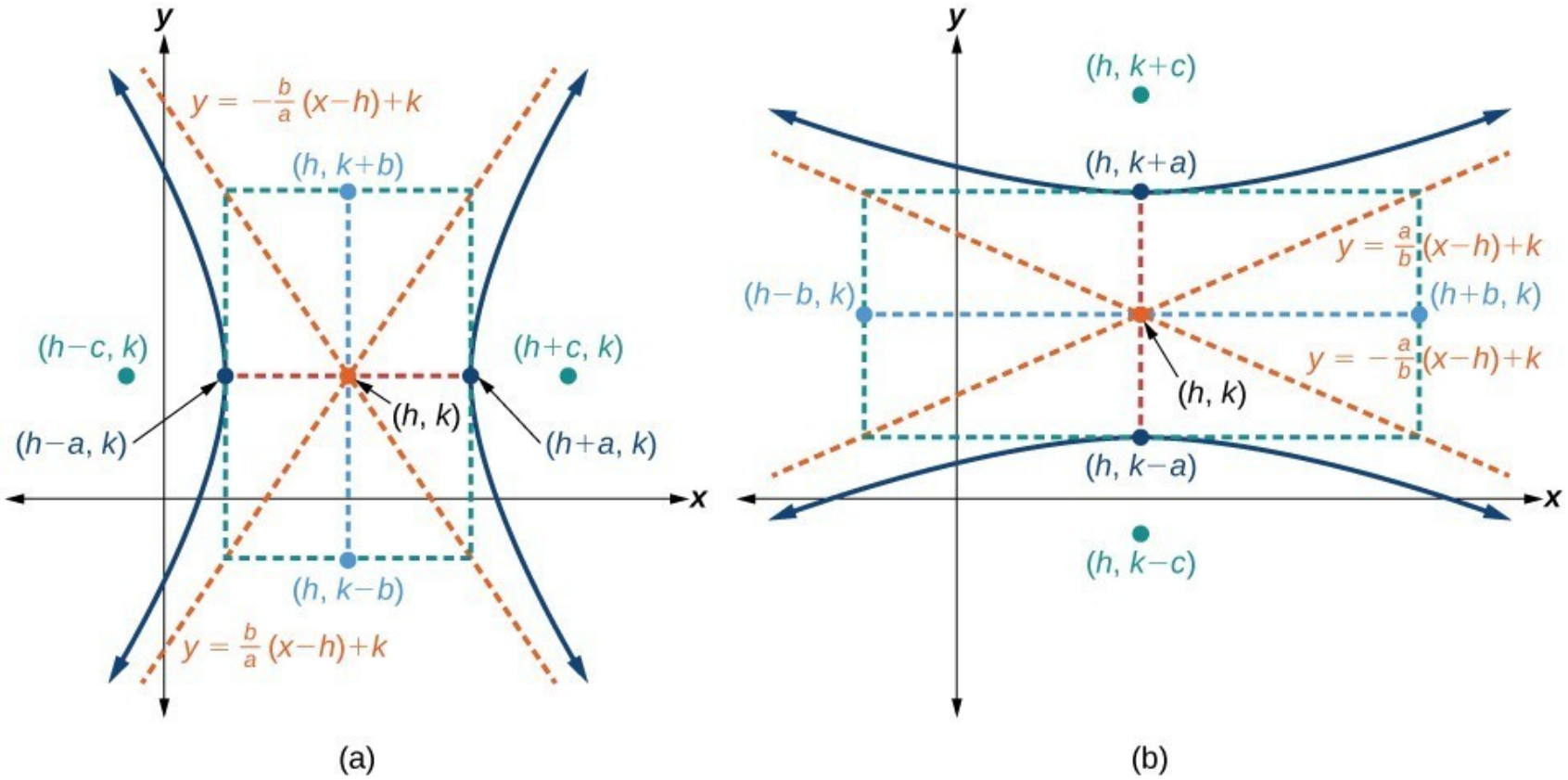
Stred $S = (m, n)$, hlavná poloos b.

Hyperbola



$$e^2 = a^2 + b^2.$$

Hyperbola

(a) $||XE| - |XF|| = 2a \Rightarrow \frac{(x-m)^2}{a^2} - \frac{(y-n)^2}{b^2} = 1$

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Hyperbola

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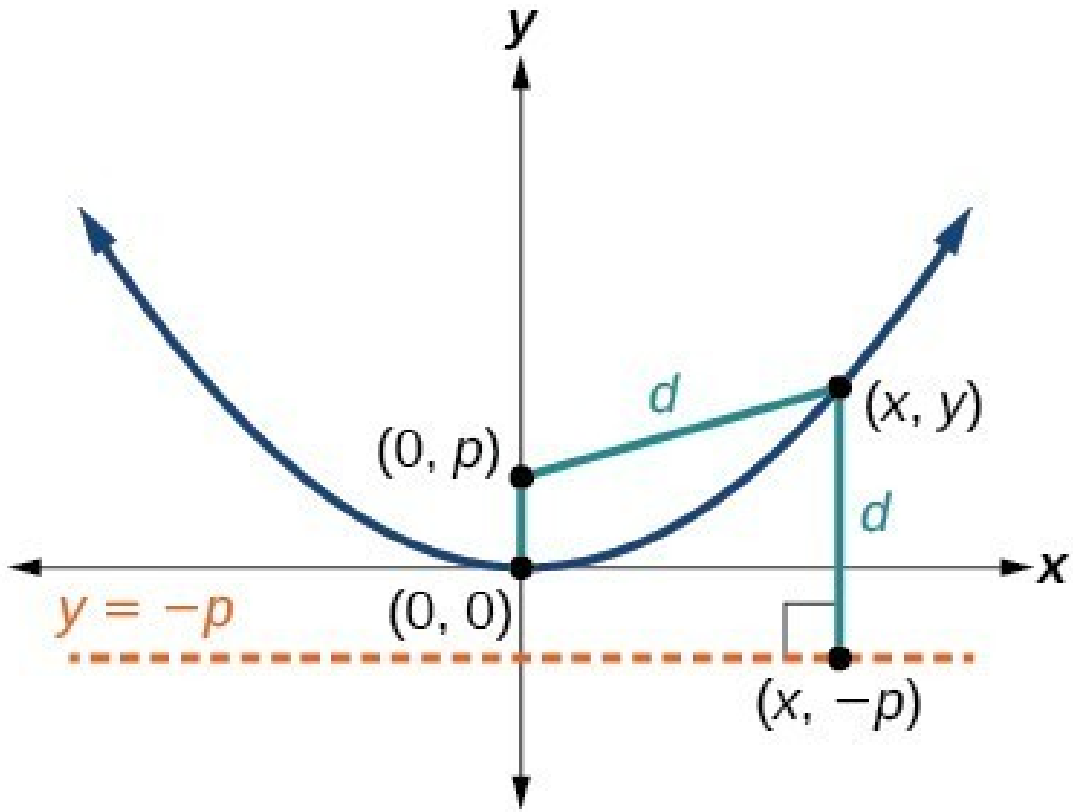
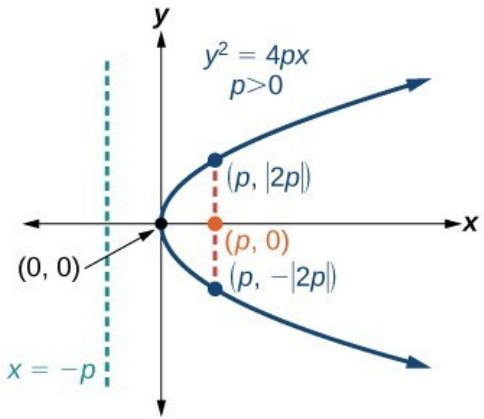
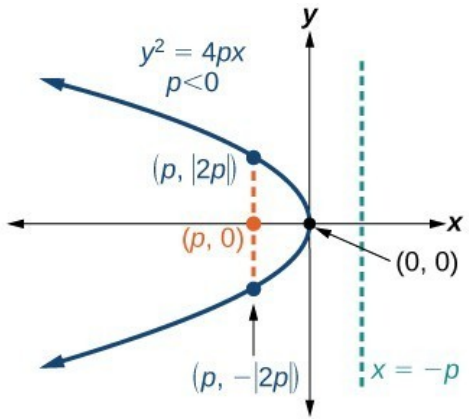
Parabola



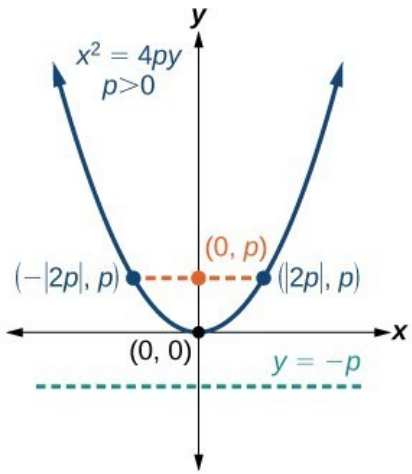
Parabola



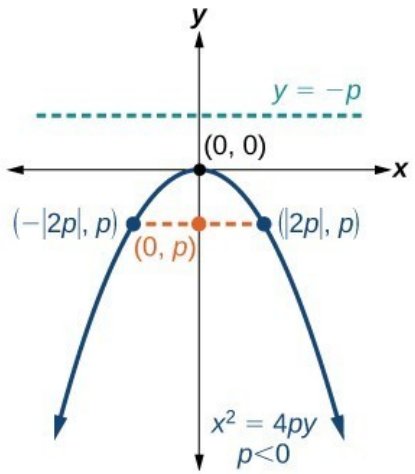
(a)



(b)



(c)



(d)

Parabola

a)–b) $|XF| = |Xq| \Rightarrow (y-n)^2 = 2p(x-m)$

c)–d) $|XF| = |Xq| \Rightarrow (x-m)^2 = 2p(y-n)$

Príklad

Nasledujúca rovnica $2x^2 + 3y^2 - 4x + 12y + 2 = 0$, určuje množinu bodov. Zistite, o akú množinu sa jedná.

Riešenie. Rovnicu upravujeme "na štvorce:"

$$2x^2 + 3y^2 - 4x + 12y + 2 = 0,$$

$$2x^2 - 4x + 3y^2 - 12y + 2 = 0,$$

$$2 \cdot (x^2 - 2x) + 3 \cdot (y^2 + 4y) + 2 = 0,$$

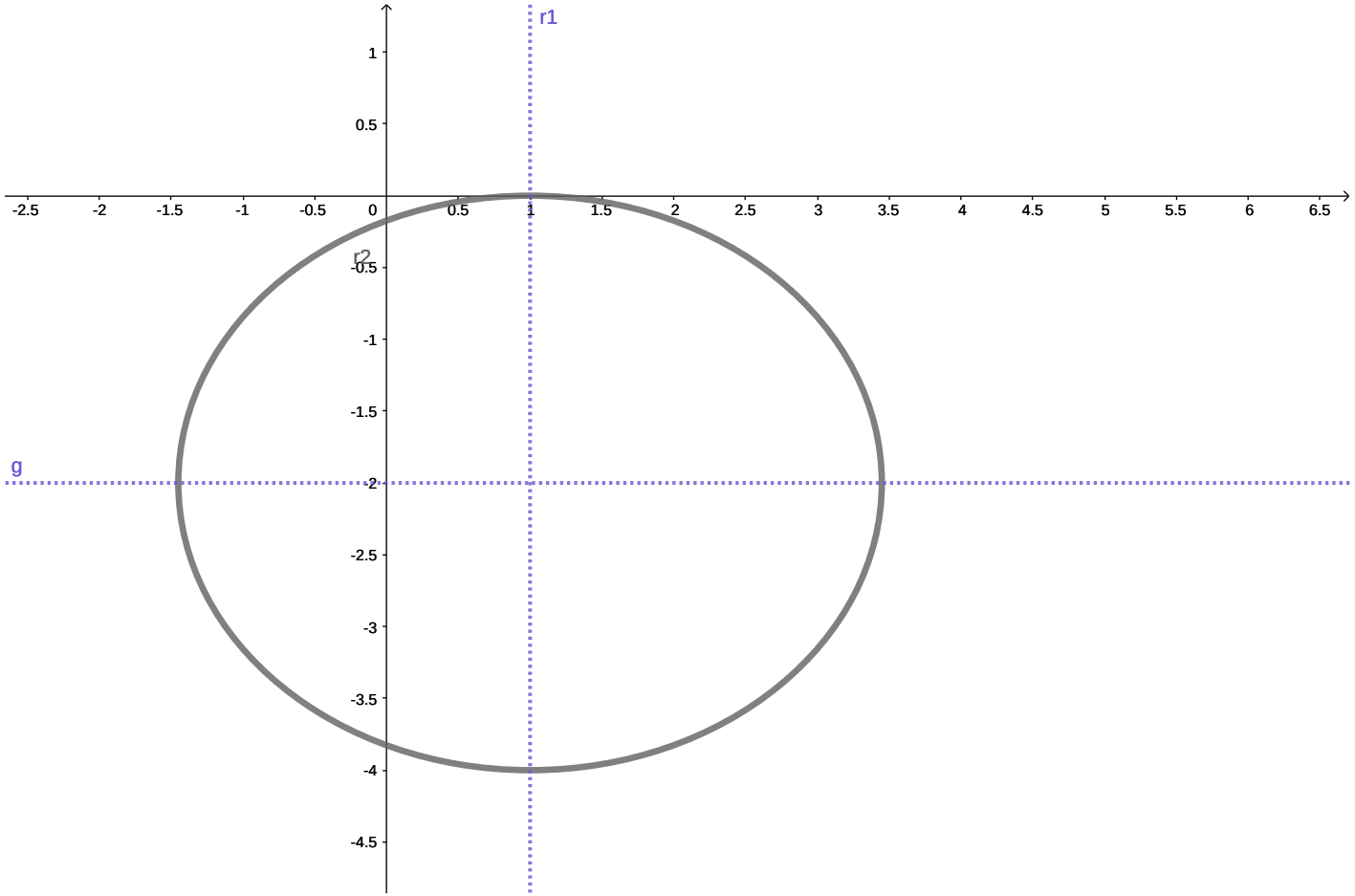
$$2 \cdot ((x-1)^2 - 1) + 3 \cdot ((y+2)^2 - 4) + 2 = 0,$$

$$2 \cdot (x-1)^2 + 3 \cdot (y+2)^2 = 12,$$

$$\frac{(x-1)^2}{6} + \frac{(y+2)^2}{4} = 1.$$

Podľa tvaru rovnice vidíme, že je to elipsa, so stredom $S = (1, -2)$, s hlavnou poloosou $a = \sqrt{6}$ a vedľajšou poloosou $b = 2$. Obrázok je na ďalšej strane.

Príklad


r1
r2
g
1
0.5
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5